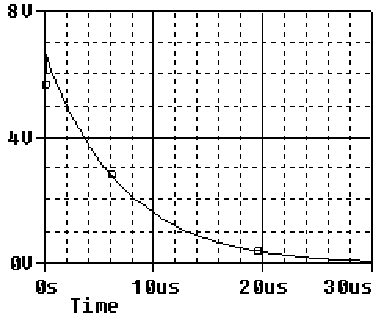
# Experiment 32: RLC Circuit Impulse Response

## Introduction

The unit impulse function, $\delta(t)$, has the properties of infinite amplitude, unit area, and infinitely short duration. This function is useful in the mathematical analysis of systems where the duration of the input excitation is much shorter than the system's response time.

A unit impulse can be scaled by the area of the excitation. This is the strength of the impulse. A rectangular voltage pulse with amplitude of 5 V and time duration of 3 microseconds has an area of 15-volt-microseconds. Its impulse function strength is 15-volt-microseconds.

A short duration exponential voltage, such as shown on the right, can be generated by applying a square wave to an RC circuit. This exponential voltage can be represented by an impulse function whose strength is equal to the area of the exponential voltage. The area can be calculated:

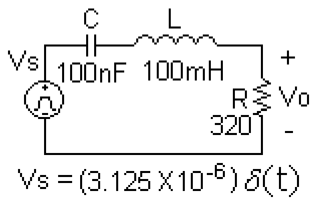



$$A = V_P \int_0^{\infty} e^{-\alpha t} dt = 6.5 \int_0^{\infty} e^{-143000t} dt = 45.5\,\mu V - \text{sec}.$$

$V_P$ is the peak amplitude at t equal to zero. The decay rate, α, is the reciprocal of its time constant. Its impulse function can be expressed as: $45.5\delta(t)$.

If this impulse function is applied to an RLC circuit whose response time is much longer than the duration of the impulse, the result of the calculation will closely approximate the actual response of the circuit.

Given that Vs in the RLC circuit on the right can be represented by an impulse function, $A\delta(t)$, the circuit's response in the s-domain is calculated as:

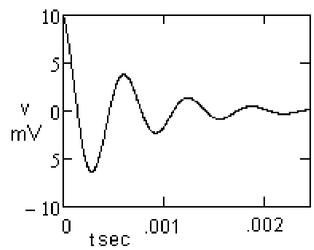



$$H(s) = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{3200s}{s^2 + 3200s + 10^8}, \qquad Vo(s) = A \cdot H(S).$$

Vo(s) is the RLC circuit's s-domain impulse response, where "A" is the strength of the impulse. In this case, "A" is the area under the voltage versus time curve of the exponential source, Vs.

Solution:

$$v = e^{-1600t}[10\cos(9871t) - 1.62\sin(9871t)]\,\text{mV}$$




Note that the circuit's impulse response is similar to its step response.

## Objectives

A circuit's impulse response may be calculated mathematically using the Dirac delta function as the input "forcing function". This lab experiment uses an exponential voltage source whose time constant is much shorter than the response time of the circuit. This source is applied to an under-damped series RLC circuit.

The response of the circuit will be compared to mathematical analysis using a Dirac delta function whose strength is approximated as the area under the exponential voltage versus time curve.

## Procedure

| Equipment and Parts |
|---|
| Function Generator, Oscilloscope, Breadboard.<br>L = 100 mH, C1 = 47 nF, C2 = 100 nF, R1 = 100 Ω, R2 = 390 Ω.<br>Rth = Function generator resistance, Rw = Inductor DC resistance. |

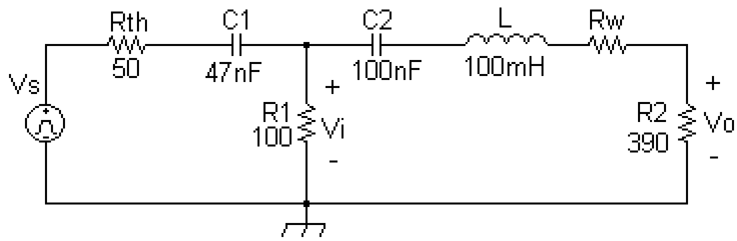
1. Measure the value of the resistors, R1 and R2, and inductor resistance, Rw.

   R1 __________ R2 ___________ C1 ______________

   C2 __________ L ___________ Rw ______________

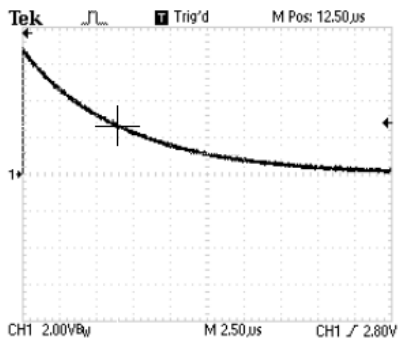
2. Connect the function generator to the oscilloscope channel 1 but don't connect it to the circuit yet. Set the function generator to produce a 200 Hz square wave, offset so the waveform goes from 0 to 10 V (no load).

3. Connect the circuit on the right. The source, Vs, and Rth represent the function generator.




4. Connect channel 1 across R1 to measure Vi. Expand the display for best accuracy.

Vi's amplitude decays to 0.37 of its maximum value in one time constant. The graph of Vi on the right shows a maximum amplitude of 7 V and time constant of 6.5 microseconds




The decay rate, α, is the reciprocal of the time constant. In this case, α is 153,800 nepers per second. The time domain equation for this response is:

$$Vi(t) = 7e^{-133800t}.$$

The strength of the impulse can now be calculated as the area under the exponential curve:

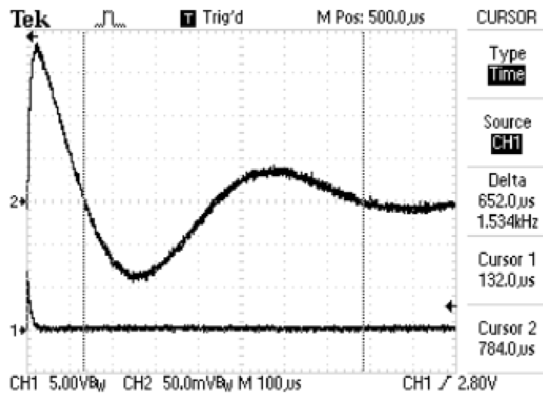
$$A = 7\int_{0}^{\infty} e^{-153800t}dt = 45.5\mu V - sec.$$

Record your results: Vi(max) _____________ α ________________

5. Connect channel 2 to measure Vo. Set the timing to 500 µS/DIV, channel 1 to 5 V/DIV and channel 2 to 200 mV/DIV. Trigger on channel 1and positive slope.

6. Set the oscilloscope to 100 µS/DIV. Adjust the oscilloscope to obtain the display on the right.

   Note the pulse on channel 1 and its timing relationship to the RLC response on channel 2.




7. Measure and record the period, T, of the damped oscillation. The display above shows a period of 652 µS. Do not use the first half cycle.

   T _______________

8. Adjust the oscilloscope so that you can measure the amplitudes and times of occurrence of the positive and negative peaks. If possible, acquire the data directly from the oscilloscope to a spreadsheet file for later analysis.

   Measure and record as accurately as possible the absolute values of the magnitudes of the positive and negative peaks and times of their occurrence (ignore the first peak near t = 0).

   $V_{P1}$ __________ $t_1$ __________ $V_{P2}$ __________ $t_2$ __________

**Analysis**

1. Use the data from procedure step 4 to calculate the strength of the impulse applied to the RLC circuit.

2. Use the result of procedure step 7 to calculate the damped oscillation frequency. Use the result of procedure step 8 to calculate the decay rate.

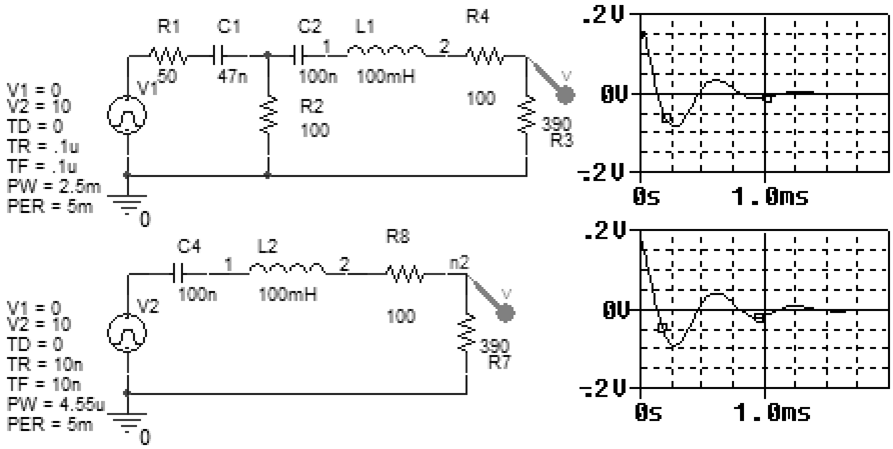
   Solve simultaneously for $V_P$ and α: $v_{P1} = V_P e^{-\alpha t_1}$ $v_{P2} = V_P e^{-\alpha t_2}$

3. Use a math program, such as *MATLAB* or *Maple*, to calculate the impulse response of the series RLC circuit using the strength of the impulse calculated in analysis step 1 above. Use your measured part values.

4. Compare the parameters of your calculated response to your experimental results. Indicate the differences in the amplitude, decay rate, and damped oscillation frequency, in percent.

5. Optional: Simulate the circuit using your part values. See example PSpice simulation below. The first circuit uses the exponential pulse generated by the RC circuit. The second circuit uses a rectangular pulse of approximately the same area. They both produce approximately the same response.

**PSpice Simulation**




**LTspice Simulation:**