Experiment 15: Network Theorems

Introduction

Thevenin's theorem states that a circuit between two nodes, a and b, can be replaced by a single voltage source (Thevenin voltage, V_{TH}) in series with a single resistance (Thevenin resistance, R_{TH}). The Thevenin voltage is equal to the "open circuit" voltage between nodes a and b. The Thevenin resistance is equal to the resistance between nodes a and b.

Norton's theorem states that a circuit between two nodes, a and b, can be replaced by a single current source (Norton current, I_N) in parallel with a single resistance (Norton resistance, R_N. The Norton current is equal to the "short circuit" current between nodes a and b. The Norton resistance is equal to the resistance between nodes a and b.

The application of Thevenin's or Norton's theorem may also require the application of other analysis methods. Determining an "open circuit" voltage or a "short circuit" current may require the use of node voltage or mesh current analysis.

Having a Thevenin or Norton equivalent circuit as a model of a more complex circuit makes it much easier to determine how the circuit will perform when loads or sources are connected to it.

It is possible to find the Thevenin or Norton equivalent of a circuit by measurement, without knowing, necessarily, the components and connections of the original circuit. If we can measure the original circuit's open circuit voltage and short circuit current, we can then determine the circuit's Thevenin or Norton resistance:

$$R_{Th} = R_N = \frac{V_{Th}}{I_N}.$$

Objectives

A bi-polar network is used to demonstrate the application of Thevenin's and Norton's Theorems. In part A, the Thevenin and Norton equivalent circuit will be determined by measurement. In part B, the Thevenin equivalent circuit will be connected and measurement results will be compared to the original circuit.

Procedure

Equipment and Parts
DMM, Power Supply, and Breadboard. Resistors: 1K, five 10K, all ¼ watt, 5%, 10KΩ trim-pot.

Part A: Bi-polar Network Circuit

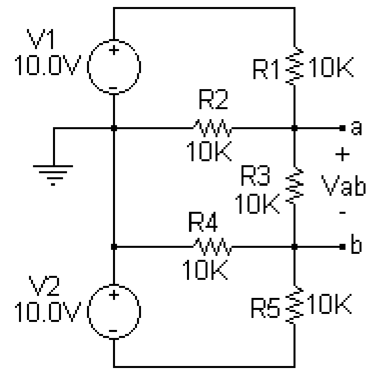
1. Measure the values of your resistors:

   R1___________ R2___________ R3___________ R4____________

   R5___________ 1K___________

2. Connect the circuit shown on the right on a breadboard. Keep track of where your 10K resistors are. Layout the circuit on the breadboard in the same way it is in the schematic diagram.

   Carefully note the power supply connections, especially that V1 is positive with respect to ground, and that V2 is negative with respect to ground.



3. Measure and record the "open circuit" voltage Vab. Measure and record Va and Vb. (Vab = V_{TH})

   Vab________ Va_________ Vb___________

5. Measure and record the "short circuit" current between nodes a and b. Set the DMM to measure current on the 20mA range. If your meter shows less than 2 decimal digits after the decimal point, use a lower current range. Connect the positive meter lead to node a. Connect the negative lead to node b. Note that Iab = I_N, the Norton current.

   Iab________________ I_N = _________________

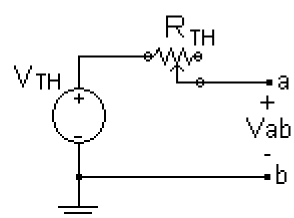
6. Calculate and record the Thevenin (and Norton) resistance: $R_{TH} = R_N = V_{TH} / I_N$.

   R_{TH}______________________

7. Connect a 1K "load" resistor between nodes a and b. Measure and record the "loaded" value of Vab.

   Vab_{Loaded} ___________________

Part B: Thevenin and Norton Equivalent of Bi-polar Network Circuit



1. Connect the Thevenin equivalent circuit of the bi-polar network using the values of V_{TH} and R_{TH} from part A of this lab exercise. Use the 10K pot for R_{TH} by setting its value to that obtained in part A, step 4.

2. Set the power supply voltage to the value of V_{TH} you measured in part A of this lab exercise.

3. Connect the 1K load resistor that you used in part A of this lab exercise to your Thevenin equivalent circuit between nodes a and b.

4. Measure and record the value of Vab_{Loaded}. Vab_{Loaded}______________________

5. Remove the 1k load resistor. Set the DMM to measure current on the 20mA range (same as in part A). Measure and record the "short circuit" current between nodes a and b by connecting the DMM between these nodes (positive lead to node a).

   Iab________________ Note that Iab = I_N, the Norton current.

Analysis, Part A

1. Write and solve the node voltage equations for Va and Vb for the bi-polar network. Be sure to use the measured values of your resistors. Calculate Vab.

2. Use a spreadsheet to compare your calculated Va, Vb, and V_{TH} to your measured values. Calculate the percent error in the measurements compared to the calculations.

3. Write and solve the mesh equations for the short circuit current, Iab = I_N, between nodes a and b. Use your spreadsheet to compare the result to your measurement by calculating the percent error. How much of the error would you attribute to meter loading effect?

Analysis, Part B

1. Use your spreadsheet to compare the measurements of Vab, Vab_{Loaded}, and Iab in part B to the measurements in part A.

2. Use your spreadsheet to compare the measurements in part B to the calculations (theoretical results) in part A.

TI-89 Example: Node Voltage Equations

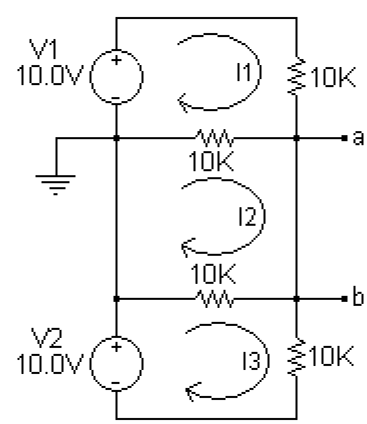
Use Va = x and Vb = y for convenience. Also, the equations below use 10,000 Ω as the resistance of all of the resistors. Use your measured values in your equations.

Solve((x-10)/10000+x/10000+(x-y)/10000 = 0 and (y+10)/10000+y/10000+(y-x)/10000 = 0,{x,y})

x = 2.5, y = -2.5 Therefore Vab = V_{TH} = x – y = 2.5 + 2.5 = 5.0V.

TI-89 Example: Mesh Current Equations

Use I1 = x, I2 = y, and I3 = z for convenience. Also, the equations below use 10,000 Ω as the resistance of all of the resistors. Use your measured values in your equations.



Loop 1: -10 + 10000 I1 + 10000 (I1 – I2) = 0
Loop 2: 10000 (I2 – I1) + 10000 (I2 – I3) = 0
Loop 3: -10 + 10000 (I3- I2) + 10000 I3 = 0

solve(-10+10000*x+10000*(x-y)=0 and 10000*(y-x)+10000*(y-z)=0 and -10+10000*(z-y)+10000*z=0,{x,y,z})

x = I1 = 1mA, y= I2 = 1mA, z = I3 = 1mA.
Therefore I2 = I_N = 1mA.